A Brief Tour of
Relativity and Cosmology

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Special Relativity, General Relativity, and Cosmology.

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Let’s cover all of it.

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In an hour.

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In an hour.

(We're going to go fast, which means we'll have to skip lots of good stuff. Even if we had a whole *day* we'd have to skip lots of good stuff. So stop me if you have questions!)

Space + Time = Spacetime

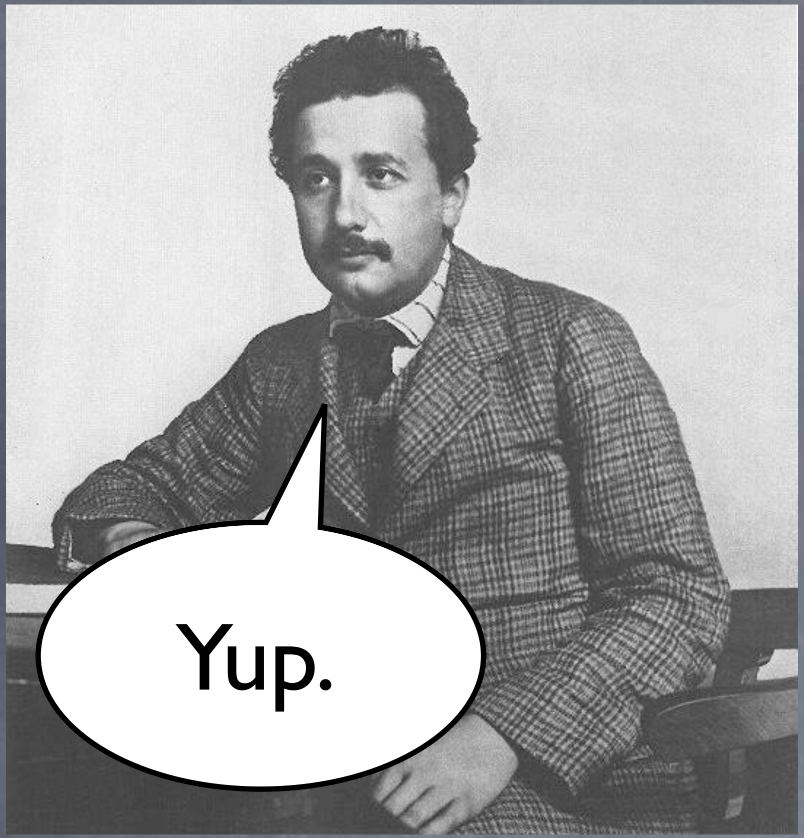
The Special Theory of Relativity

It’s 1905, and Einstein has trouble reconciling three widely-accepted “facts”.

1. Electric and Magnetic fields are described by Maxwell’s equations. They predict that light propagates at about 300 million m/s.

2. The same laws of physics should describe the observations of two people moving at constant relative velocity.

3. Velocities are added and subtracted in the “usual” way. If I’m driving east at 60 mph and you’re driving west at 60 mph, we see each other approaching at 120 mph. Right?



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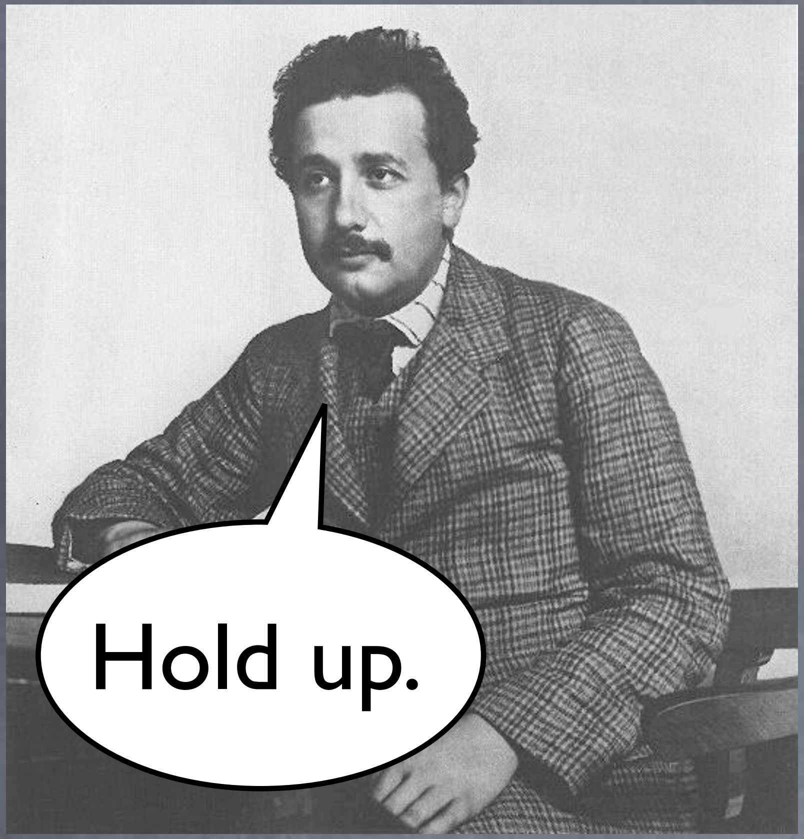

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This is a problem! It implies that moving observers should see different speeds of light, contradicting points 1 and 2!

Einstein responds by replacing "Galilean" relativity. He changes the rules for how two people moving at a constant relative velocity compare their descriptions of physical events.

Two observers moving at constant relative velocity disagree on *where* an event happens, and also *when* it happens.

$$\bar{x} = \frac{x - v\,t}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \bar{t} = \frac{t - \frac{v}{c^2}\,x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Tricky, but it's the only way for the two people to agree on the predictions of physical laws!

Einstein's new theory – *The Special Theory of Relativity* – has many important consequences.

Length Contraction

I see you move by at speed $v=0.8c$. You shout "Look at this $1\mathrm{m}$ long ruler!" I say "Wrong! It's only $0.6\mathrm{m}$ long!"

$$L = \sqrt{1-\frac{v^2}{c^2}} \times 1\,\mathrm{m}$$
$$= \sqrt{1-(0.8)^2} \times 1\,\mathrm{m} = 0.6\,\mathrm{m}$$

Time Dilation

I notice that your watch is ticking too slowly! After 3 seconds pass on your watch, my watch has already ticked off 5 seconds!

$$T_{me} = \frac{T_{you}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{3\,\mathrm{s}}{\sqrt{1-(0.8)^2}} = 5\,\mathrm{s}$$

These are real phenomena, supported by mountains of experimental evidence.

Have a phone with GPS? You can test SR right now. GPS satellites emit time-stamped signals that your phone uses to triangulate your position. To get your location to $\pm 5\mathrm{m}$, you need to time the arrival of signals with $20\mathrm{ns}$ accuracy. But the satellites move at ~ $3900\mathrm{m/s}$ relative to you:

$$T_{you} = \frac{T_{sat}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$T_{sat} = (86{,}400\,\mathrm{s})\sqrt{1 - \left(\frac{3.9 \times 10^4}{3 \times 10^8}\right)^2} = 86{,}399.999993\,\mathrm{s}$$

Over the course of one day the satellite's clock "falls behind" your phone's clock by ~ $7\mu\mathrm{s}$. This would cause a $2\mathrm{km}$ drift in your position, if your phone didn't account for it!

Special Relativity forces us to recognize that space and time aren't distinct things. Sure, each observer has their own notion of space and time, but they are mixed up compared to everyone else's. The only way to make sense of this is to realize we live in a four-dimensional *Spacetime*.

The geometry of Spacetime is a little different than what you are accustomed to. It leads to phenomena like length contraction and time dilation. These effects are only revealed by very large velocities or very sensitive measurements, which is why they hadn't been noticed previously!

> *"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."*
>
> *— Hermann Minkowski, 1908*

Rockets, Gravity, and Geometry

The General Theory of Relativity

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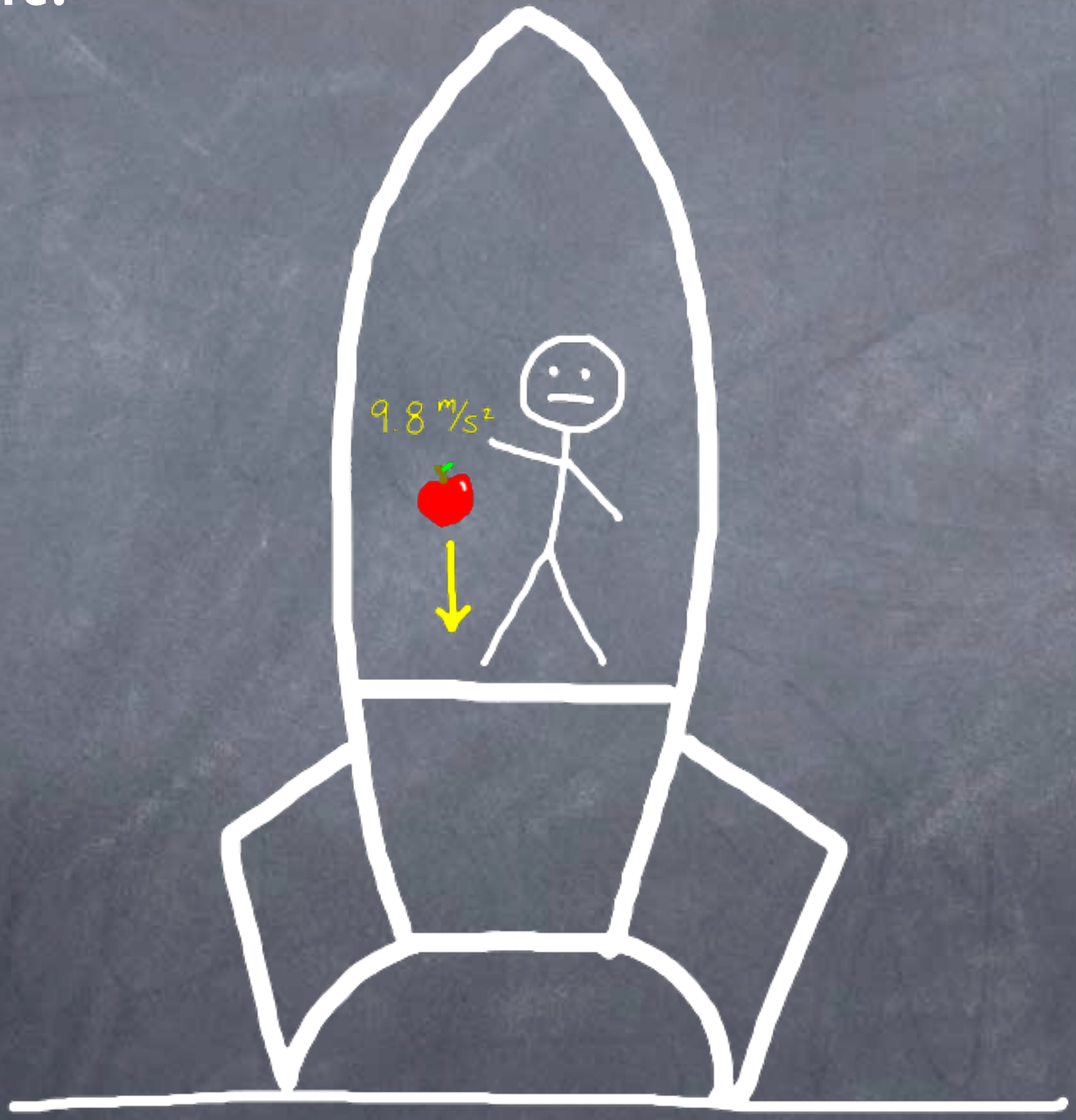
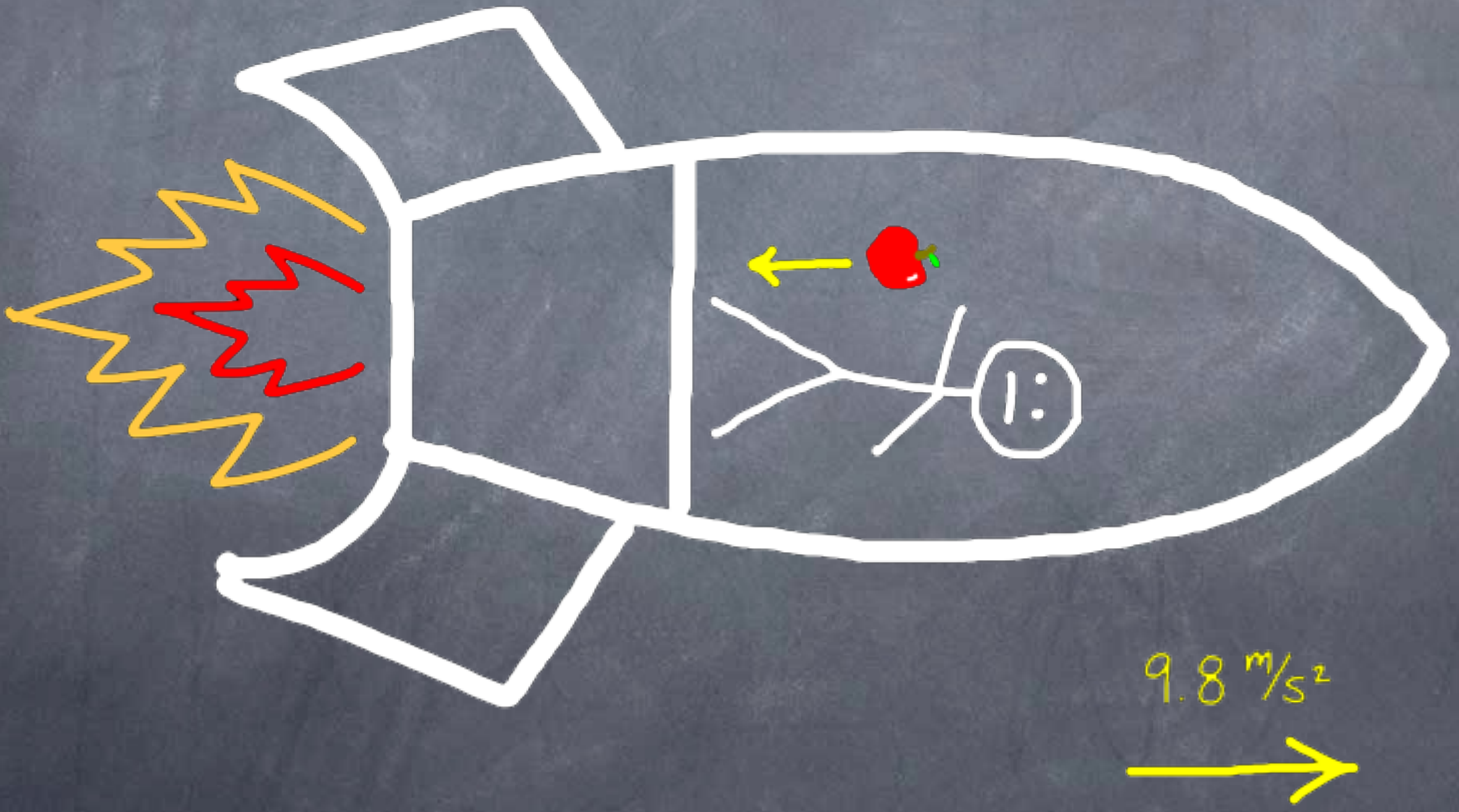
Here is the thought that occurred to him.

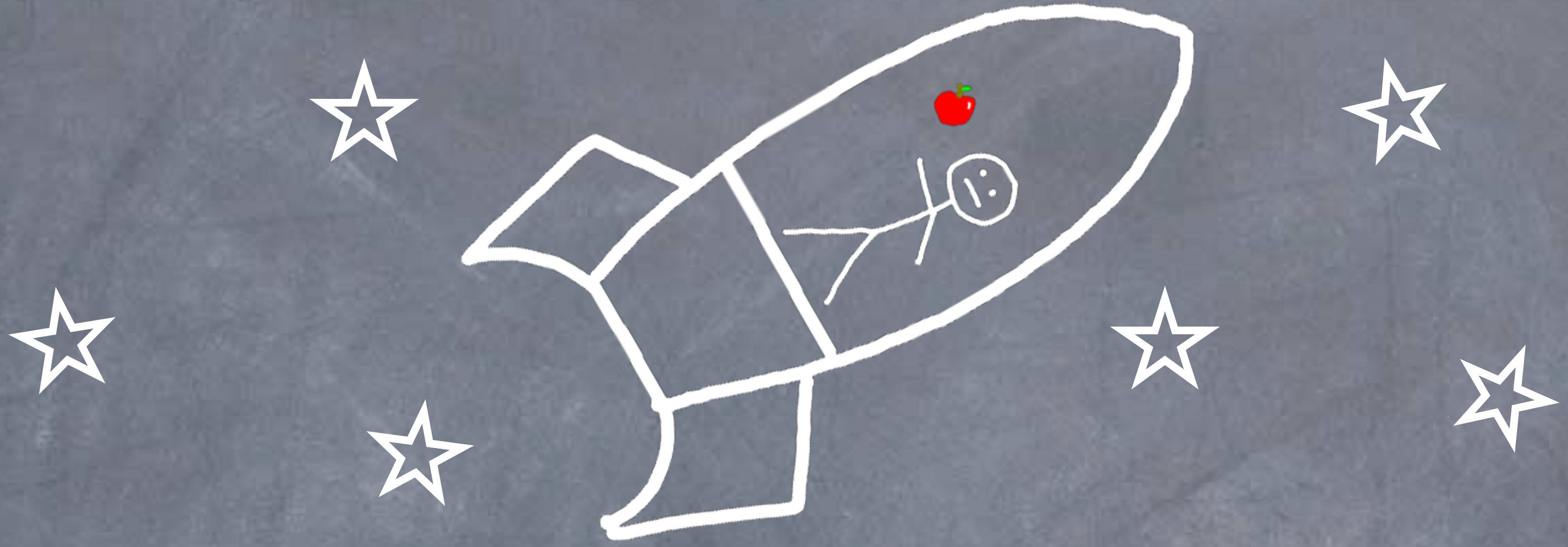
You are sitting in a rocket, in a cabin with no windows. You let go of an apple, and it accelerates towards the floor at a rate of $9.8\text{m}/\text{s}^2$. A-ha! Obviously you are still sitting on the launchpad. Right?



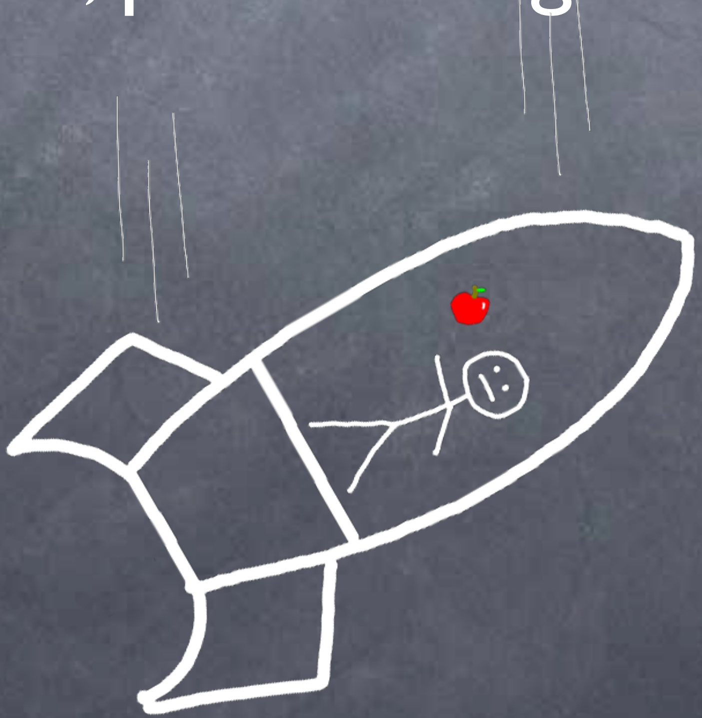
Are you sure? The same thing would happen if you were already in space, accelerating along a straight path at a rate of $9.8\mathrm{m/s^2}$. Since there are no windows in your cabin, you can't tell which situation you are in!



What if you were experiencing weightlessness? Then you must be floating in space, far from any sources of gravity. Right?



Not necessarily! You'd experience the same thing if the rocket was in free fall, plummeting towards Earth.

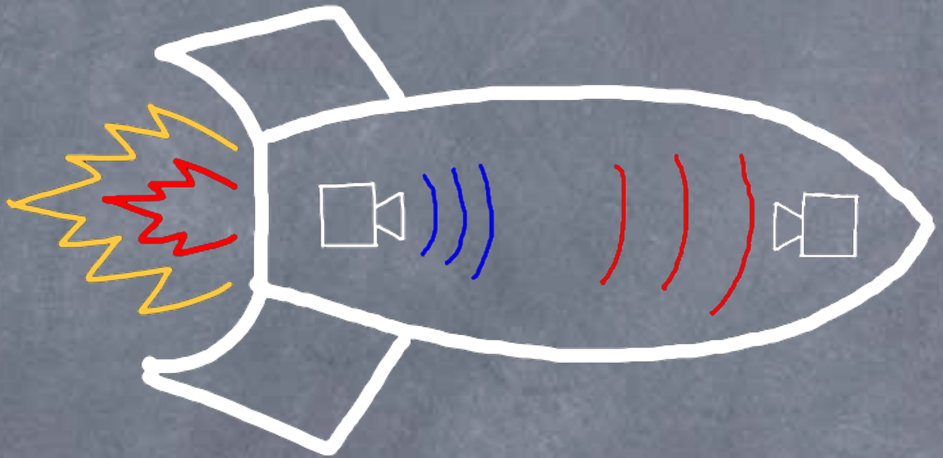


The point is that *locally* – there inside your small cabin, over short periods of time – you can't tell! Locally, no experiment can distinguish between an accelerating rocket and a gravitational field. Likewise, there's no difference between free fall and the lack of a gravitational field.

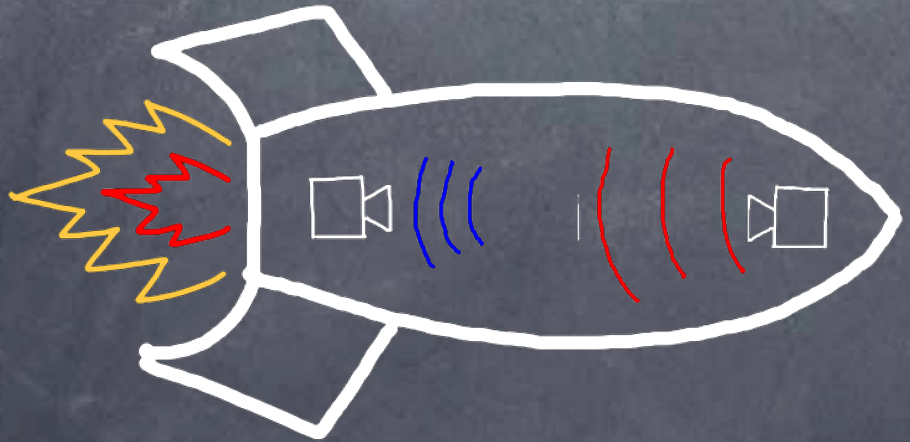
Of course, once you can see outside your cabin or do an experiment that lasts long enough, you can tell a difference. But in that case you are relying on more than *local* information.

This is an important part of what we call "The Equivalence Principle". The key conclusion (for our purposes) is that any physical effect caused by an acceleration must also be produced by a gravitational field, and vice-versa.

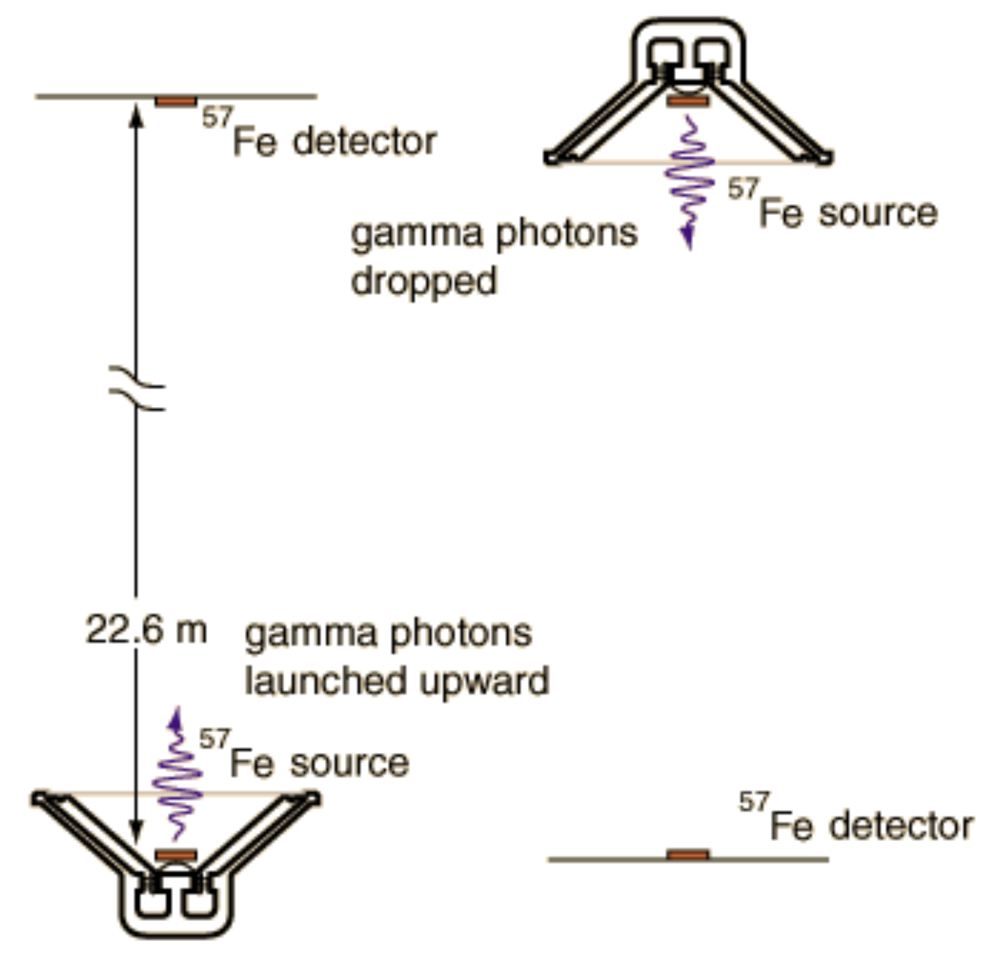
For example, a signal sent from the back of an accelerating rocket towards the front will be *red-shifted* – the frequency will decrease.



If you send a signal from the front to the back, it will be *blue-shifted* – the frequency will increase.



A gravitational field produces the same effect! This was verified at Harvard by Pound and Rebka in 1959.



Photons moving up experience a gravitational redshift.

Photons moving down experience a gravitational blueshift.

This isn't a trick, and it's not limited to certain kinds of "clocks". It's a physical statement about the passage of time in a gravitational field. As you get closer to a gravitating body, time passes more slowly than it would far away.

Suppose you're hanging out near a spherical body – like the Earth – with mass M and radius R. (We'll ignore the fact that Earth rotates.) Then the rate at which two clocks tick depends on their distance from the center:

$$\frac{(\Delta T)_1}{\sqrt{1-\frac{2\,M\,G}{c^2\,r_1}}}=\frac{(\Delta T)_2}{\sqrt{1-\frac{2\,M\,G}{c^2\,r_2}}} \qquad r_1, r_2 > R$$

In fact, this effect is even more important for GPS than the Special Relativistic effects we discussed earlier!

$$M_E = 6.0 \times 10^{24}\,\mathrm{kg} \qquad R_E = 6.4 \times 10^{6}\,\mathrm{m}$$

$$G_N = 6.7 \times 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2} \qquad r_{sat} = R_E + 2 \times 10^{7}\,\mathrm{m}$$

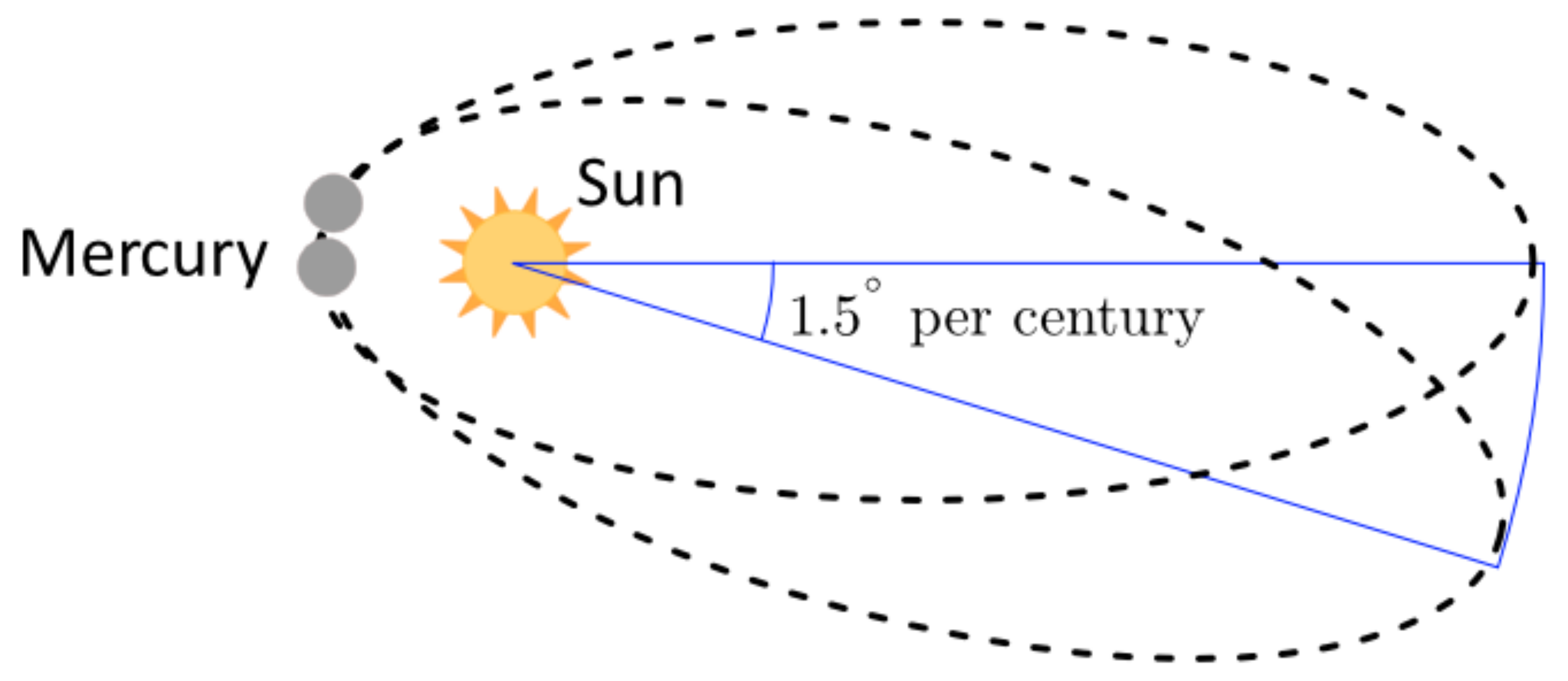
$$\rightarrow \quad (\Delta T)_{\mathrm{surf}} \simeq \left(1 - 5.3 \times 10^{-10}\right) (\Delta T)_{\mathrm{sat}}$$

After one day, the GPS satellite's clock would be ahead of ground-based clocks by about 45μs – it runs faster because it is further from the gravitating body. This would cause locations to "drift" by around 10km every day.

The physicists and engineers that developed the GPS system adjusted the atomic clocks onboard the satellites, slowing them down to account for this effect!

Einstein's insights led him to develop the *General Theory of Relativity*.

General Relativity (GR) makes important predictions about phenomena that aren't explained by Newtonian gravity. For instance, it explains a small but measurable discrepancy in the orbit of Mercury.



Newton's theory could not account for 43 arc-seconds of the precession (about one-hundredth of a degree per century). GR gets the orbit exactly right!

In General Relativity, the spacetime of Special Relativity is *curved* by the presence of matter and energy. (The surface of a sphere, or the shape of a hilly countryside are good, but not perfect, analogies.) The motion of matter and energy through spacetime is affected by this curvature. To paraphrase the physicist John Wheeler:

> Matter and energy tells Spacetime how to curve,
> and the curvature of Spacetime tells matter and
> energy how they should move.

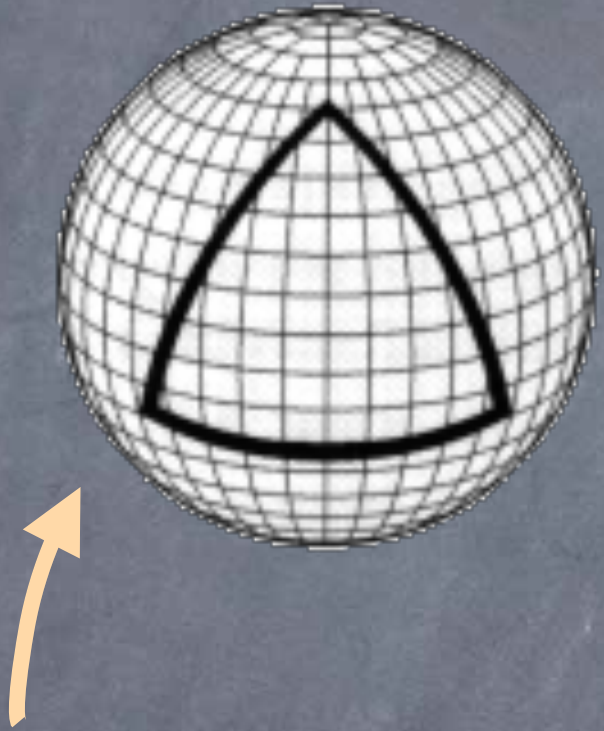
The effect of matter and energy on the geometry of spacetime is wrapped up in a deceptively simple equation:

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = 8\pi\, G\, T_{\mu\nu}$$

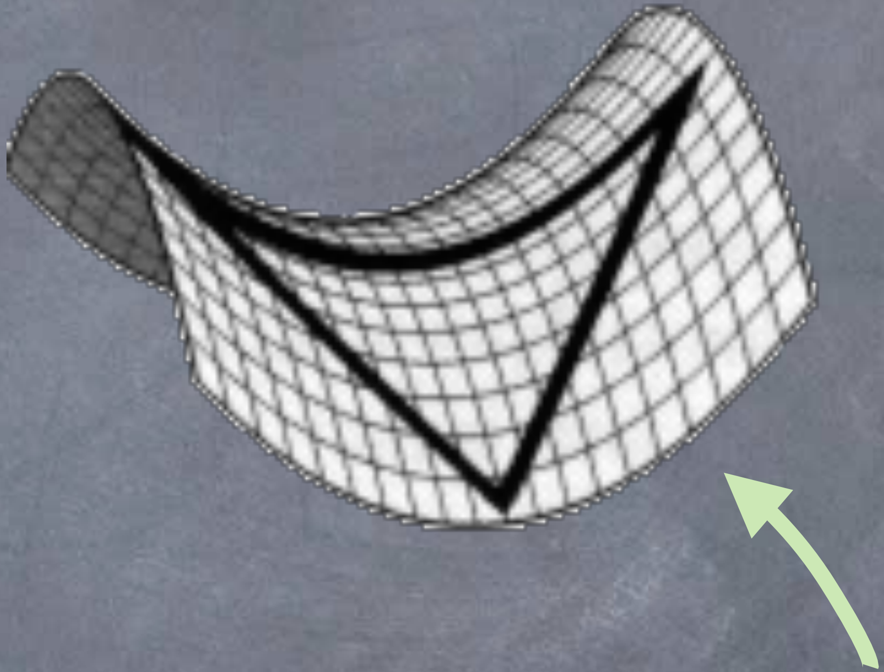
Curvature of
Spacetime over here

Matter and Energy
over here

Gravity is a manifestation of the curvature of spacetime. Freefall – the apparent absence of gravity locally – is just motion along what passes for a straight line in the curved spacetime. These paths are called "geodesics".



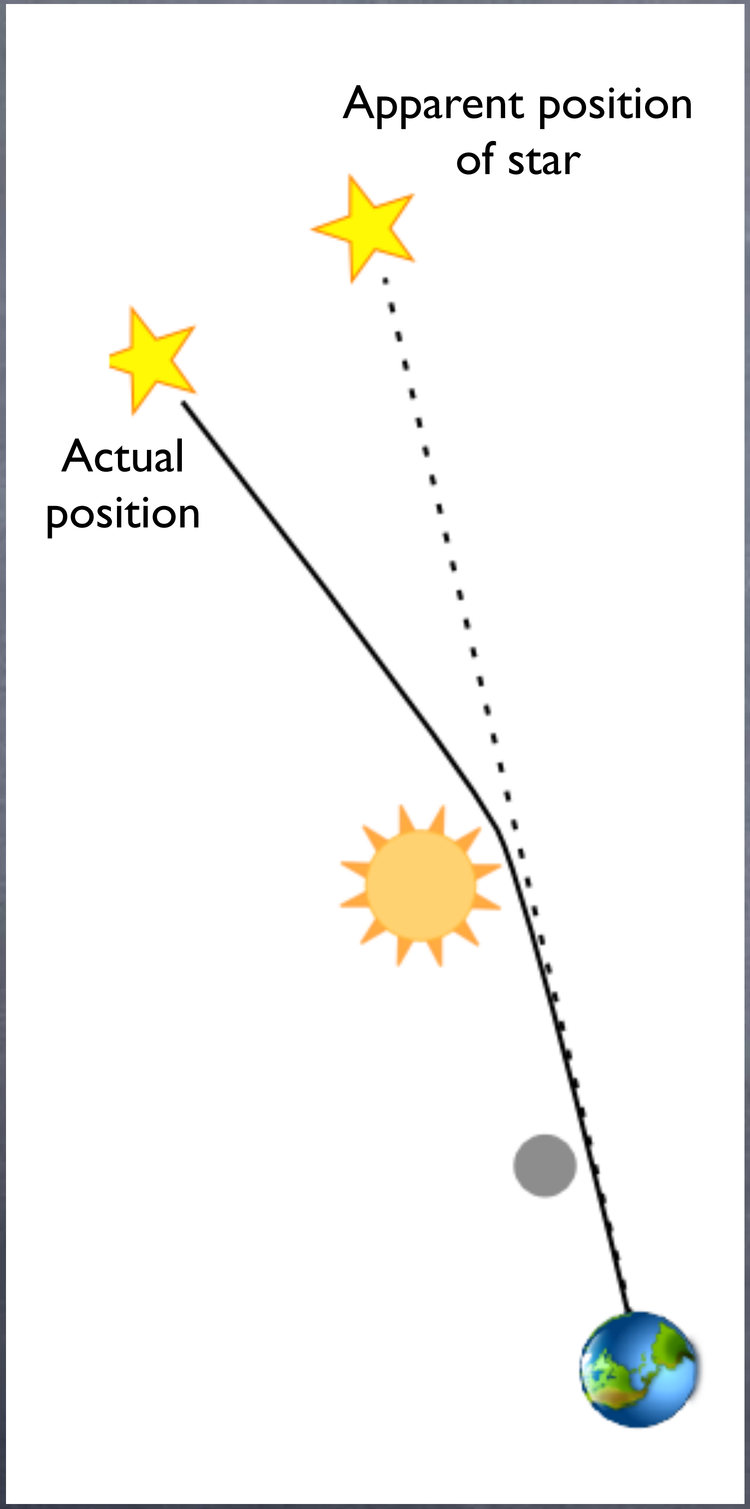
Positive Curvature

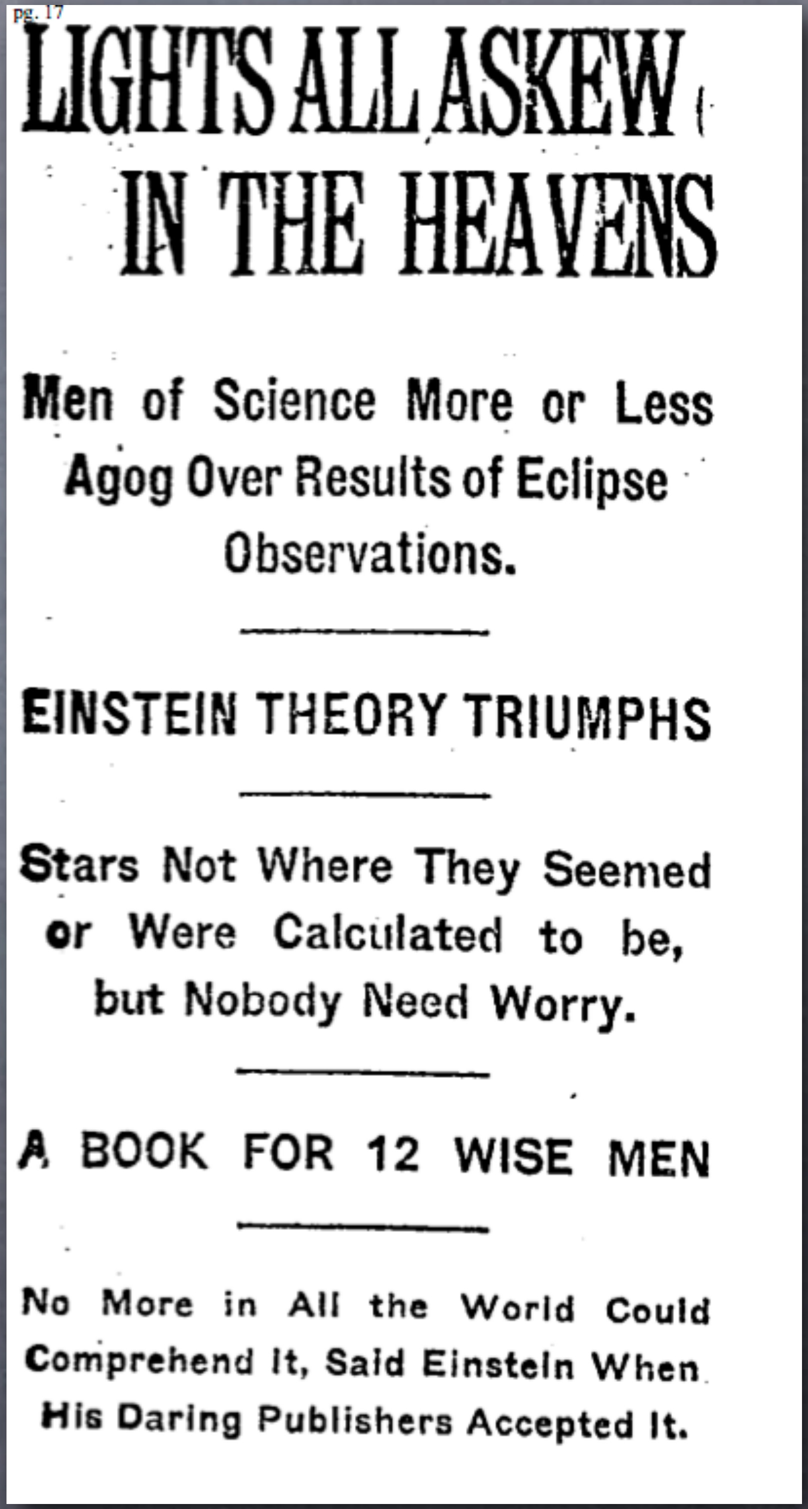


Negative Curvature

Spatial curvature is a little easier to visualize than temporal curvature. When the "time" part of spacetime is curved, you should think of that as affecting the rate at which clocks run.

The mass of the Sun curves Spacetime in such a way that the geodesic followed by light from a distant star is "deflected". Einstein predicted this effect, which is very small, and it was observed during the 1919 eclipse.

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LIGHTS ALL ASKEW IN THE HEAVENS

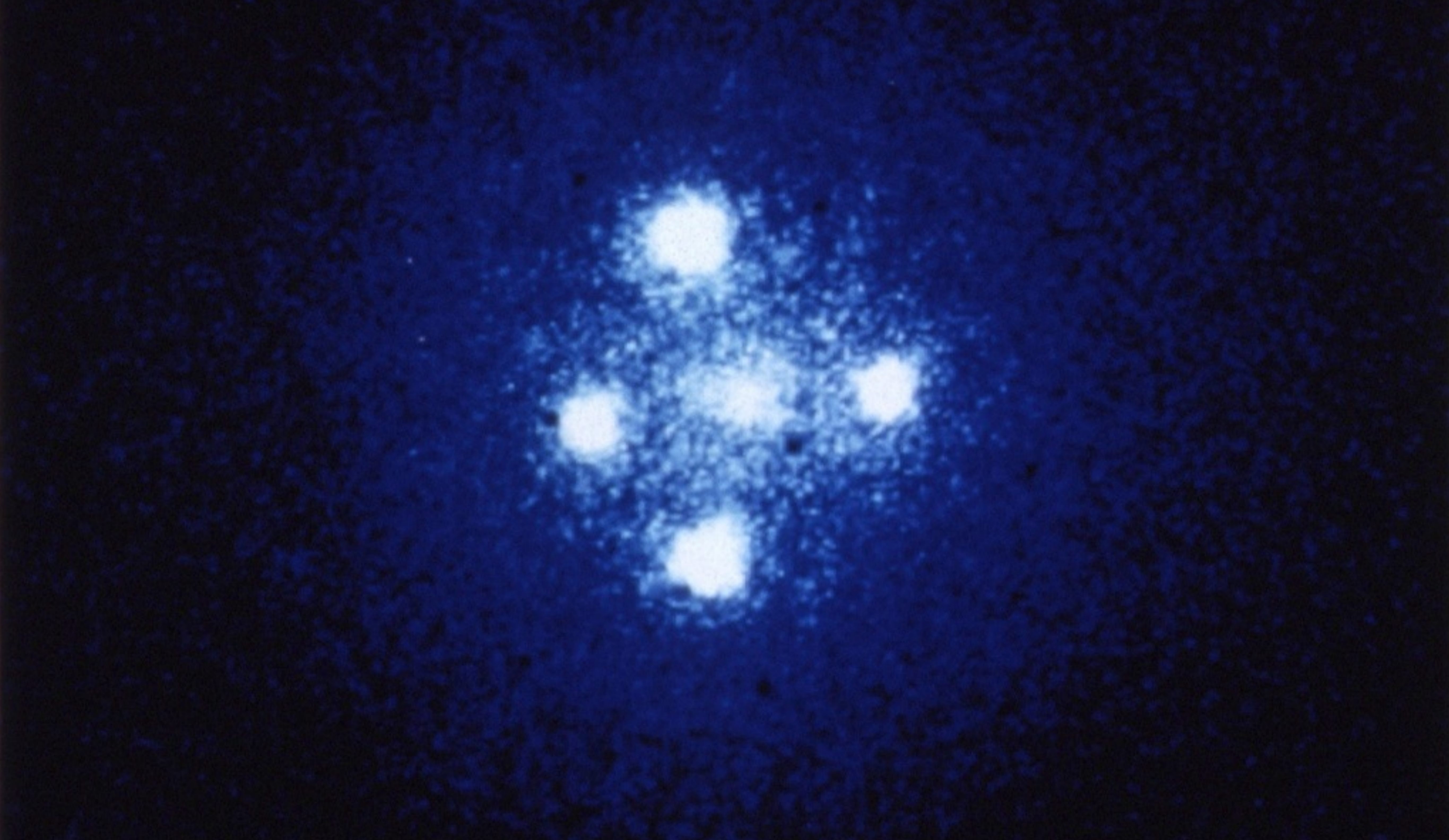
Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

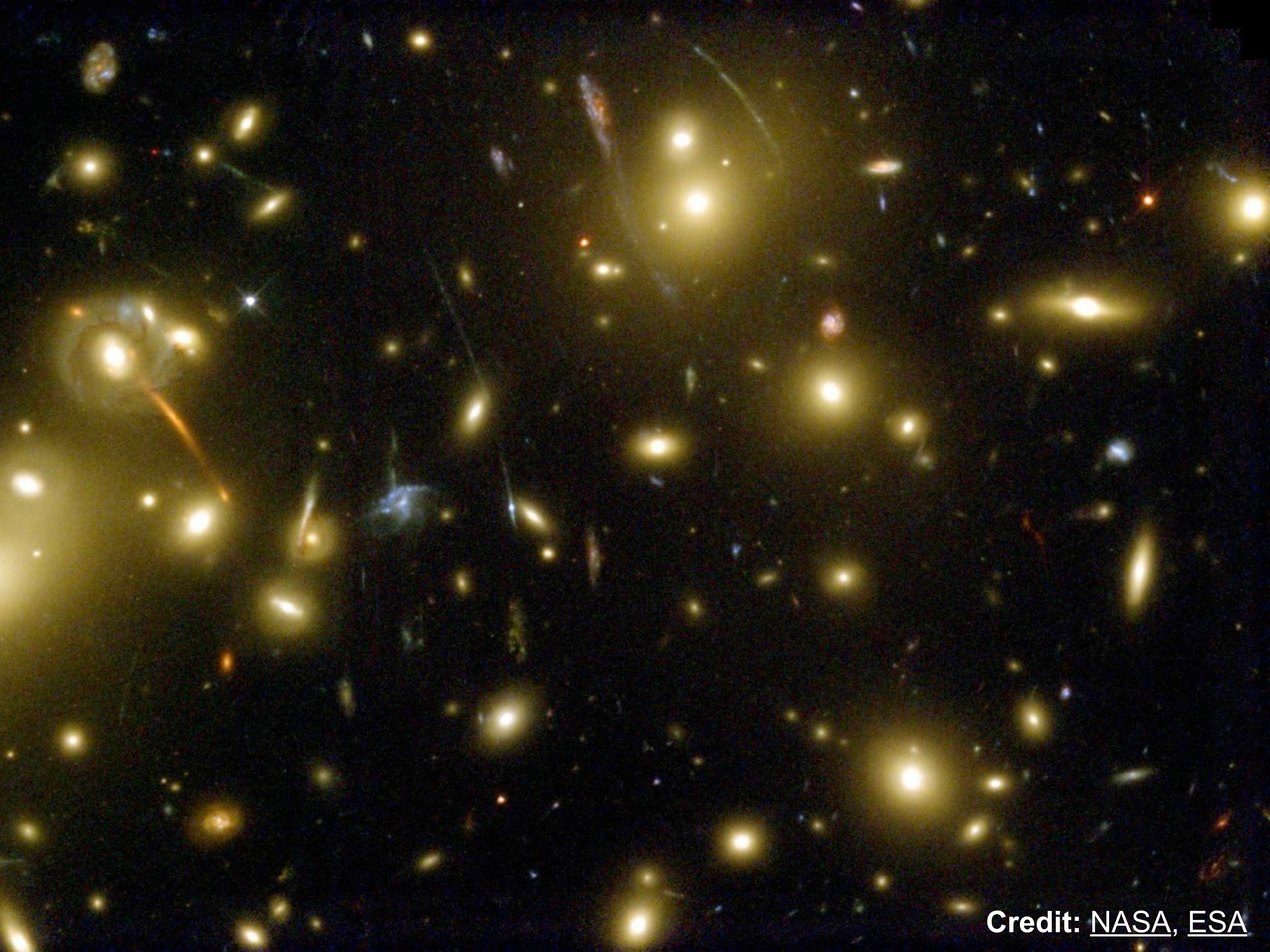
A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.



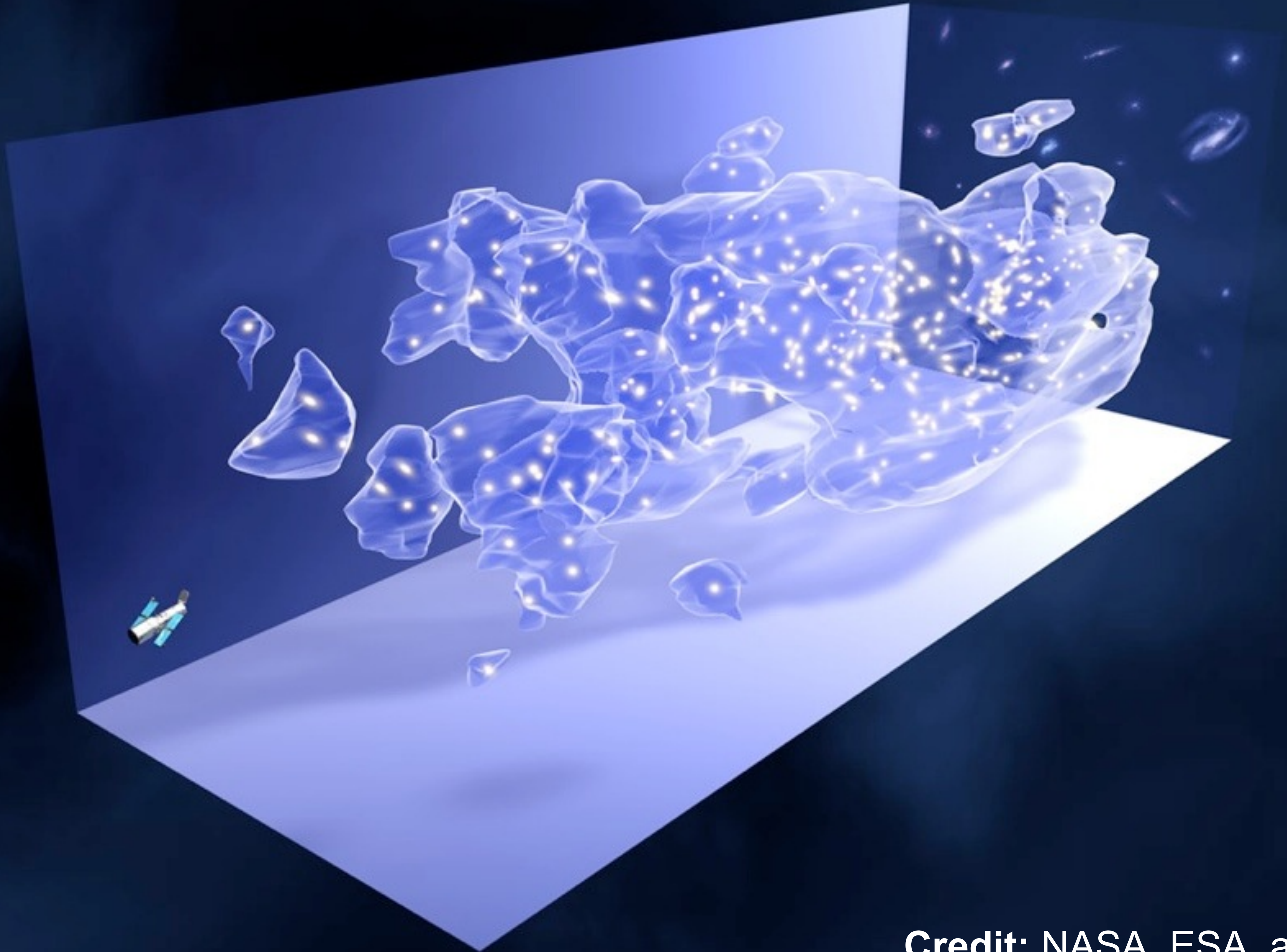
More generally, the deflection of light due to gravitating mass is known as *Gravitational Lensing*. This is an example of "strong" lensing, where the effect is so dramatic that multiple images appear.

Credit: NASA, ESA, and STScI



Credit: NASA, ESA

In “weak” lensing, the effect is small and must be recovered through a careful statistical analysis. It is one of the main tools that astronomers use to map the distribution of matter in the Universe.



Credit: NASA, ESA, and STScI

There's a lot more we could say about General Relativity. We're not even scratching the surface!

We haven't talked about Black Holes, or Gravitational Waves. If the other physicists knew I was skipping Black Holes they would probably kick me out of the club.

(If you have questions about these things we can talk afterward!)

The Once and Future Universe

Cosmology in the 20th and 21st Centuries

In 1924, the astronomer Edwin Hubble discovered that "spiral nebulae" are in fact separate galaxies.

Fig 25 Herschell
51 Messier, sketched
April 1845, carefully
compared with
original on different
nights but no
micrometer employed.
Handed round the
section at the
Cambridge meeting

Sketch of a spiral nebula by a 19th century astronomer, the Earl of Rosse.

A few years later, in 1927, Hubble showed that these galaxies tend to be moving away from us! For really close galaxies the effect is obscured by other kinds of motion, but the recession velocity increases with distance. This is Hubble's law:

$$v = H_0\, d \qquad H_0 \simeq 70\,\frac{\mathrm{km/s}}{\mathrm{Mpc}}$$

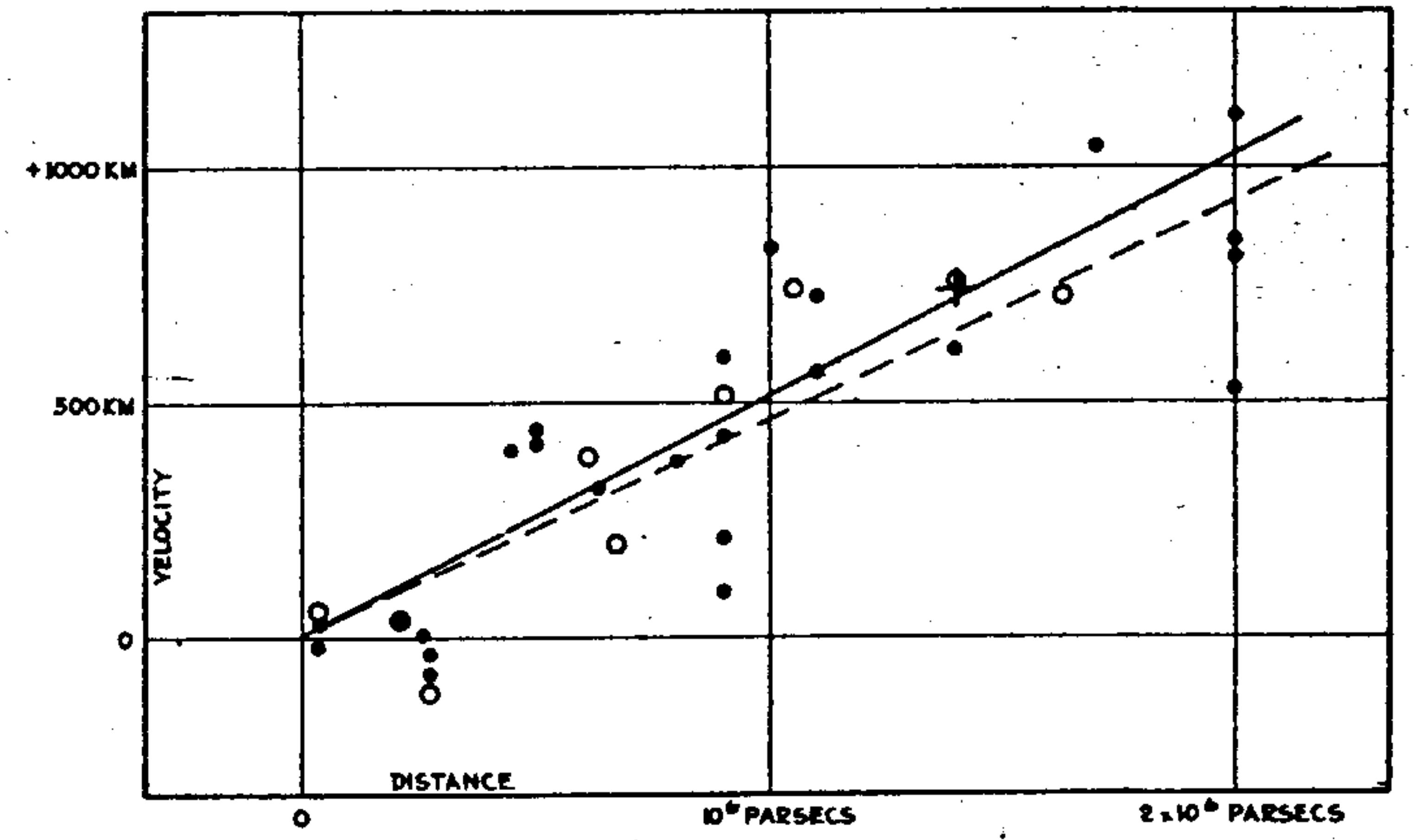


FIGURE 1

Hubble saw this effect everywhere he looked in the sky. Why would galaxies in all directions be receding from us?

Actually, the galaxies aren't moving away from us. They're all moving away from each other! How can this be?

> *The Cosmological Principle:* On large scales, the Universe is roughly homogenous and isotropic. Its properties at any point are the same as at any other point (homogeneity), and it looks the same in all directions (isotropy).

The Cosmological Principle is an assumption – a starting point for our models – that constrains them in testable ways. Combined with Hubble's Law, it suggests that on large scales everything is moving away from everything else.

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This is only possible if the Universe itself is expanding.

This is an incredible conclusion! Of course, Einstein had reached it much earlier. He applied GR to Cosmology in 1917, but he kept arriving at a Universe that changed over time – either expanding or collapsing.

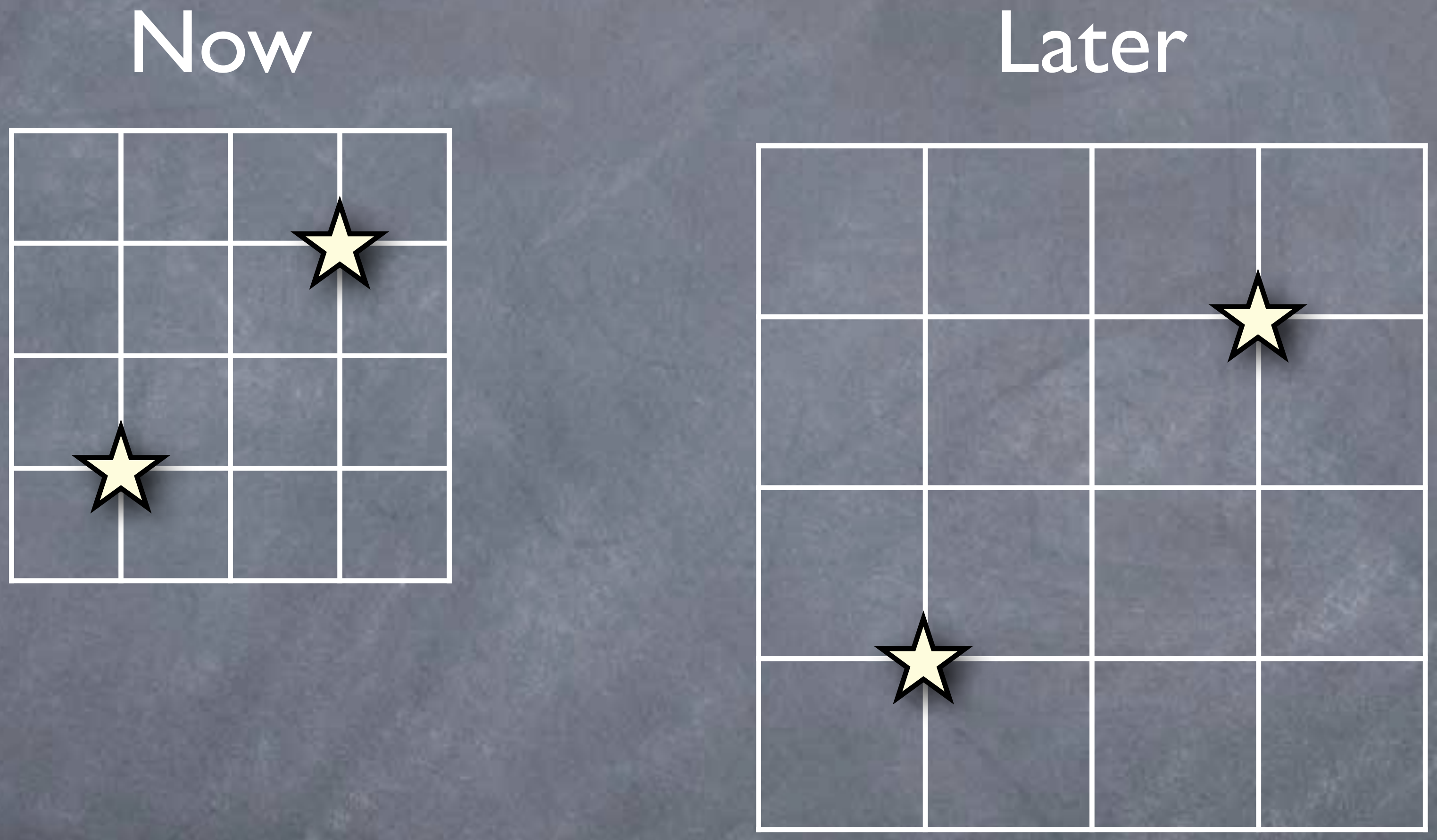
Einstein thought this was unlikely, so he tried to fix his result by adding a new term to his equations that made the Universe static and unchanging. The new term represented a gravitating energy intrinsic to space itself, and not tied to matter or radiation. It was dubbed the "Cosmological Constant".

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - g_{\mu\nu} \Lambda$$

Hubble's conclusions convinced Einstein that his original models were correct. He renounced his modification, calling it his "biggest blunder".

But this is Einstein, so even his mistakes have a way of being correct...

How do we model the Universe on large scales? We set up coordinates to describe where things are. *Peculiar velocity* represents a change in an object's coordinate position. But even if there is no peculiar motion, the physical distance changes over time due to *Expansion velocity*.



$$d_{\rm phys} = a(t)\, d_{\rm coord}$$

The "scale factor"

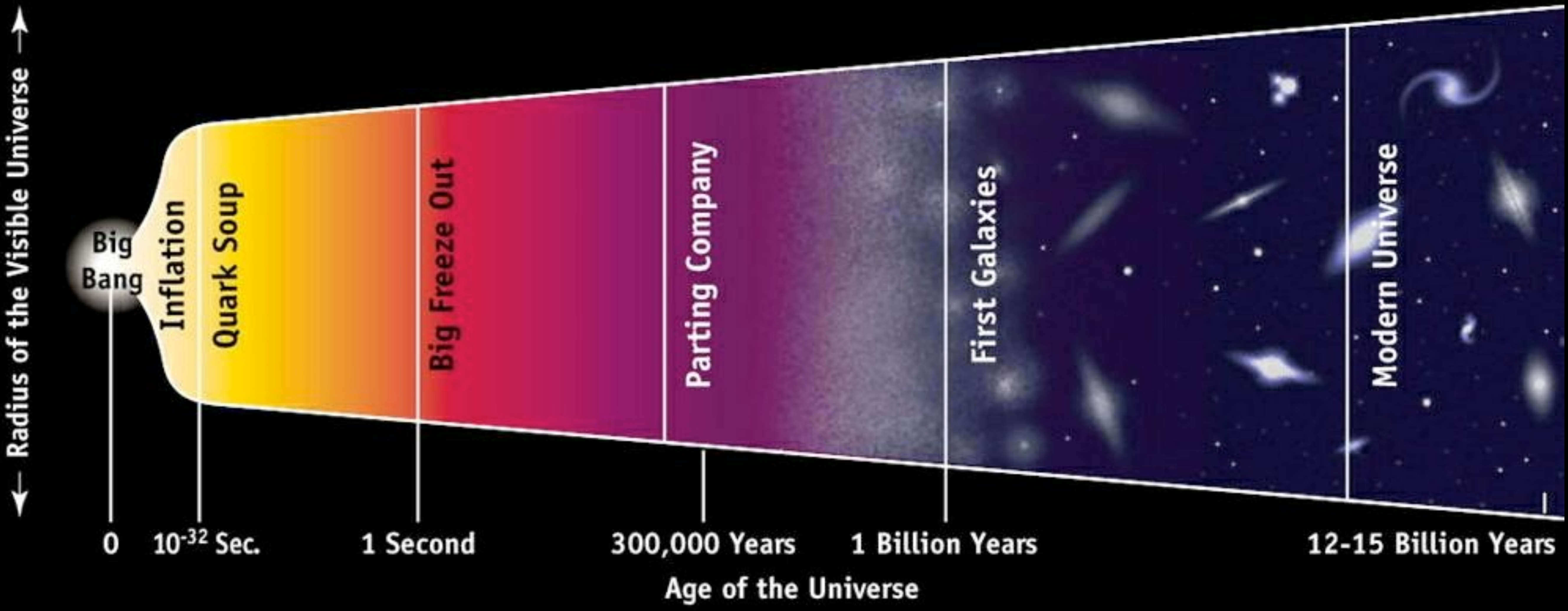
Once we apply the Cosmological Principle, GR gives us two equations that describe the effect of matter and energy on the scale factor:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left(\rho_{\rm matter} + \rho_{\rm rad} + \ldots\right)$$

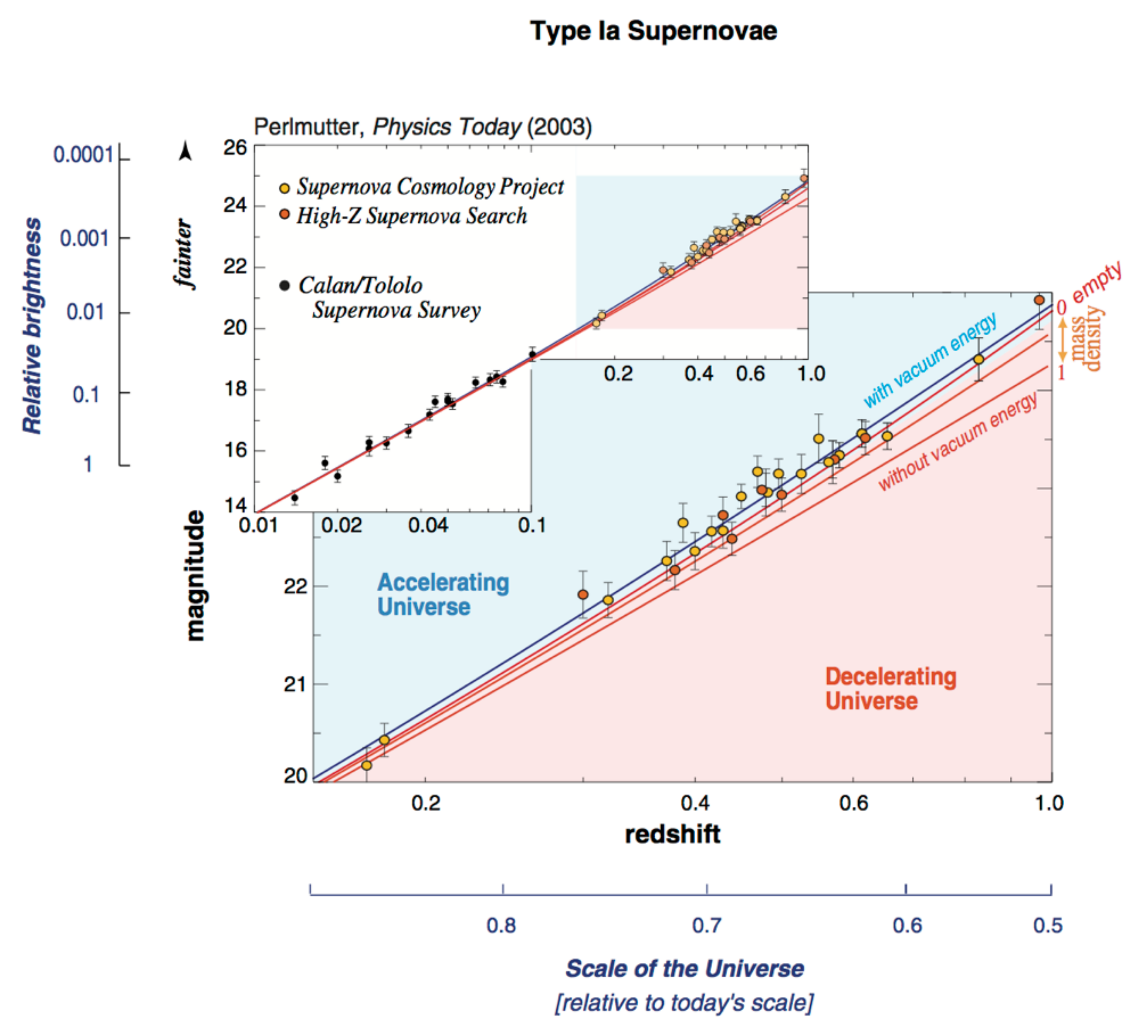
$$\frac{\ddot{a}}{a} = -4\pi G \left(p_{\rm matter} + \frac{1}{3}\rho_{\rm matter} + p_{\rm rad} + \frac{1}{3}\rho_{\rm rad} + \ldots\right)$$

Much of 20th century Cosmology was an attempt to measure the amount of "stuff" on the right-hand-side of these equations!

If you follow the evolution of the Universe back in time you find that the scale factor shrinks. Conditions become hotter and more extreme, and at early enough times the contents of the Universe are so energetic that we don’t know enough physics to say what happens. This “moment” is called the *Big Bang*. We have a very detailed understanding of what happens starting not too long after this.



But cosmologists got a major surprise in 1997. In a survey of distant supernovae, the exploding stars appeared too dim!



After a number of corroborating experiments ruled out other explanations, there was only one possible interpretation: the Hubble Expansion of the Universe is *speeding up*.

I don’t think anyone expected this. Everyone thought that the matter and energy content of the Universe was slowing down the expansion.

No one knows exactly why this is happening! The best guess is Einstein’s “mistake.” A Cosmological Constant – the same mechanism that kept Einstein’s early models from collapsing in on themselves – is driving the expansion. It fits the data very well, and makes up about 74% of the “stuff” out there.

(By the way, those same experiments show that about 20% of the Universe is some sort of “Dark Matter” that we also can’t explain.)

Maybe this isn’t such a blunder after all! In fact, the other major development of 20th century physics – Quantum Mechanics – predicts a Cosmological Constant.

Unfortunately, QM predicts too much Cosmological Constant — between 60 and 120 *orders of magnitude* too much. Imagine trying to answer a question and getting it wrong by a factor of

1000000000000000000000000000000
0000000000000000000000000000000
0000000000000000000000000000000
0000000000000000000000000000

Something goes very, very wrong when we try to apply our two best physical theories to the same problem.

In my opinion this is the deepest and most profound problem facing modern physics. GR and QM are the most accurate and best-tested theories ever devised. And in those rare situations where they are both relevant, everything goes bonkers!

Luckily, the theories don't have too much overlap. It happens in Cosmology, and again when you study Black Holes, but the problems I'm describing don't really interfere with day-to-day physics.

Then again, the things that bothered Einstein didn't affect everyday physics, and they led him to Special Relativity! It's the same thing here: something must be subtly wrong with GR or QM or both. What could it be?

I wish I could tell you what the answer is. But I don't know! Physicists don't know the answer to *lots* of interesting questions!

1. Why is the Cosmological Constant so small?
2. What happens to the stuff that falls into a Black Hole?
3. Is Quantum Mechanics really as random as it seems?
4. How do we make General Relativity and Quantum Mechanics compatible with each other? What has to change?

The story of fundamental physics in the 21st century will be about *your* attempts to answer these questions.

Thanks!
Questions?

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